The Spatial Analysis of Fractal Geometry in Mughal Architecture: A case of Wazir Khan Mosque Lahore, Pakistan

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***Abstract*: Geometry has played very substantial role for Mughal architecture in various spatial and ornamental systems. The distinguishing features of Mughal architecture have always been ordered repetition, radiating structures and rhythmic patterns. This paper focuses on the fractal geometry which has the same attributes with reference to design and urban planning. The three salient features of fractal geometry such as similarity, iteration and change in scale are the main characteristics of the Mughal structures. The purpose of this study is to study the occurrence of fractal geometry in Mughal architecture in two and three dimensional features Mughals used ornamental patterns which are close to the nature and its rules to show their thoughts of unity in diversity. The core objective is to investigate the relationship between the fractal geometry and historic architecture of Mughals with decorative symbolic expressions. The case study of Wazir Khan Mosque Lahore represents the use of fractal geometry in the Mughal architecture and measurement of fractal dimensions through Box Counting Method also inclined towards the existence of fractals. Therefore, assuming the existence of fractal geometry in Mughal architecture is a possible and realizable issue and it deserves research and analysis.**

***Keywords*: Fractal geometry, Fractal Dimensions, Box Counting Method, Mughal Architecture**

I. INTRODUCTION

Geometry is one of the oldest sciences known by the man. The linear geometry has a key role to develop architectural shapes, volumes and forms. In the late twentieth century, a new type of geometry which deals with the non-linear and irregular shapes was introduced and it termed as fractal geometry. During the last few decades, researches and architects started to study fractal geometry with reference to architecture and urban planning. Benoit Mandelbrot was the pioneer of the word "fractal" and he introduced this term during the late 1970's [2]. Yessios attempted to explain the role of fractal geometry to develop architectural shapes. Jenks gave the idea of self-similarity in fractals explaining the concept of unity and diversity through different scales. Another researcher, Boville described the mathematical method to calculate the fractal dimensions of the historical building facades. He also studied the relationship between the parts and whole within the structure. . On the other hand, researchers Durmisevic, Eaton, Lorenze, Kue-Chung, Cupo, and Almuqqarum discovered methods to use fractals in architectural design and reached the clarification of self-similarity and the repetition of the same shape. Moreover Zangana's study dealt with the fractal rhythm in the heritage building facades [13].

There is a strong relationship between art, architecture and mathematics. Although, it is difficult to find interconnections between fractals and architecture. This paper investigates to find similarities between attributes of fractals and characteristics of Islamic architecture. Geometry and geometrical patterns in Islamic architecture are critical issue that should be given attention to prove the occurrence of fractals representing the objects in nature. Geometry has played a vital role to develop architectural shapes and volumes through imaginable objects in nature. Moreover, comprehensive system of Islamic architecture, especially in mosques and palaces couldn't be studied without the concept of geometry. The effort to do this will result in chaos and collapse [13], [15].

Fractal geometry is a new branch of geometry introduced few decades ago Application of fractal geometry in Wazir Khan Mosque can help to understand the philosophy of design principles in Mughal designs and decorative elements. A deep analysis of Plans, Elevations and geometric patterns in ornamentation two and three dimensional will prove that fractals or non-Euclidian geometry also exist along with Linear or Euclidian geometry.
To achieve the goal of the research, we will present some of the concepts of the fractal geometry as well as applying them to the study of the architectural designs in general and in the Mughal architecture in particular.

II. FRACTALS & NON-FRACTALS

Mandelbrot defined a "fractal" as any curve or surface that is independent of scale. Fractals are that part of geometry which deals irregular shapes. As an example, objects in nature can have the fractal properties which are different from non-fractals. The Euclidian geometry can't explain the geometry of the objects existed in nature such as branching of a tree, clouds, coastline etc. The geometry of these objects is much closer to the fractal geometry and they possess the attributes which are different from the linear geometry. Although, architectural shapes and forms are based on the linear geometry but we can find fractal components in architecture too at micro and macro levels [2], [4], [5].

A. Self-similarity

The properties of the fractals are different to that of non-fractals. When we enlarge non-fractals, we find no further details as shown in Fig 1.

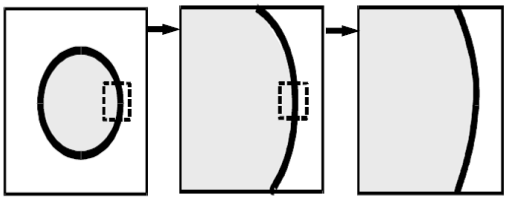

Fig. 1: Properties of non-fractal

On the other hand, when we enlarge fractals, we come across the further geometric details of that object resembling the properties of the objects found in space and nature as shown in Fig 2.

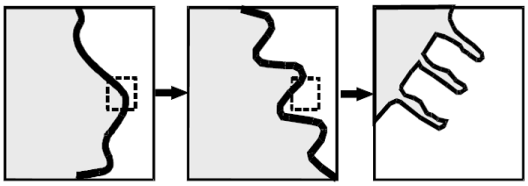

Fig. 2 : Properties of fractals, self-similarity

The study of the fractal properties enlighten our thoughts with the idea that there must be some principle which is applicable to the shapes which are not straight rather they are irregular and non-linear.[4] A fractal object is self-similar, never ending phenomena where repeated elements change in scale but retain a similar shape. According to the Mandelbrot, any portion of the fractal object if blown up in scale would appear identical to the whole curve. It means transition from one scale to another can be termed as iteration of a scale process [6]. The self-similarity can be observed in the arches of Mughal buildings such as Taj Mahal and Hammayon's Tomb [4].

B. Scaling

A non-fractal object has most pieces that are about the same size.

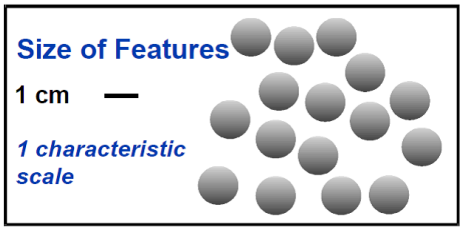

Fig. 3 Properties of non-fractals (Scaling)

A fractal object has pieces of all different sizes. The variation in the size of the pieces of fractal objects is much larger than the variation in the size of the pieces of non-fractal objects. Typically, there are a few big pieces, some medium-sized pieces, and very many tiny pieces as shown in Fig. 4.

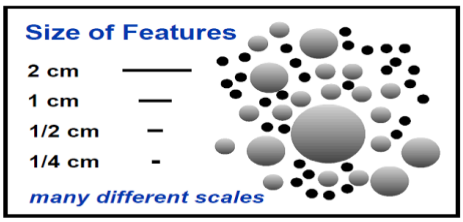

Fig. 4 Properties of Fractals (Scaling)

III. FRACTAL DIMENSION

Fractal shapes has many important characteristics, one of them is fractal dimension. This character explains that fractal dimension is not an integer but have a range between 1 and 2 whereas Euclidian geometry deals with the integers. Mandelbrot found that fractal shapes can be described through a real number which has its value between 1 and 2 and this explains the meander and complexity and it is termed as "Fractal Dimension" (13). Fractals Dimensions can be understood with the help of mathematical equations. If on multiplying the length of a line by 2, we get two lines but on multiplying length and width of the square with 2, we get 4 copies of the original. In case of cube we get 8 copies of the original. If we write number of copies or the

magnification factor as a power of 2 and the exponent will be the dimension (5). Then n=2 d, here 2 is the number of self- similar pieces

$$\text{Magnification factor} = (\text{Number of self- similar pieces})^{\text{Dimension}}$$

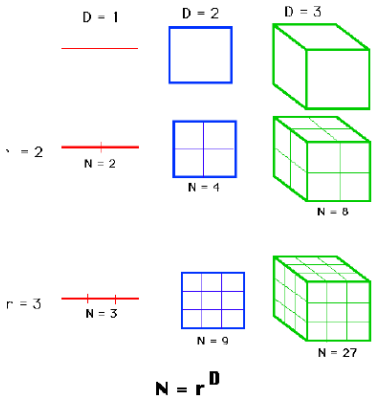

Fig. 5 Dimensions

To understand the concept of fractals or objects in space, we have to study two dimensions. One is self-similarity dimension (Ds) and other is Box-counting dimension (Db.). All these dimensions are directly related to the Mandelbrot dimensions (D) which says that fractals are not integers as there value ranges between 1 and 2.

A. Fractal Dimension

There are different methods to measure the fractal dimensions such as Cantor's Comb(1872), Helge von Koch's Curve (1904), Sierpinski's Triangle (1915), Gaston Julia Sets (1917) but the most graphical method for the approximate calculation of the fractal dimension is the box (grid) counting method because of its low computer processing requirements. This method is recommended as one of the suitable methods to yield an idea of the fractal dimensions [5]. This method is based on the research of W.Lorenze (2002) and C. Bovil (1996). They proposed Box-counting dimension method to check the level of fractality in architectural forms [16] . A Mughal building Taj Mahal is one of those and they proved that fractal dimensions are not integars but they bears the values between one and two. In this research paper, an analysis of the fractal dimensions will be done on the bases of same method in Wazir Khan Mosque Lahore, Pakistan.

B. Box Counting Method

The Box Counting Method to calculate the fractal dimensions in the buildings is very popular because of its low computer processing requirement. The value of the fractal dimension decides about the details in the form of the buildings. Higher the value of the fractal dimension, more details in the form. It shows that the fractal dimension of any form describes the progression of the details.

The Box Counting Method is as follows

a. Select the form which has been selected to measure the fractal dimension and place a grid on size "S1"over the top of it.
b. Count all the occupied Boxes "C1".
c. Now double the size of the grid "S2" and again count the occupied boxes "C2".
d. Repeat the process
e. Tabulate the results as per table I

Table 1 : Tabulation of Results

Sr. No.	Grid Size	Occupied Boxes Count
1.	S1	C1
2.	S2	C2
3.	S3	C3

f. The fractal dimension "D" will be determined with log-log plot S1 Vs. C1 across the scale 2 to 1
g. D (2 to 1) = log C2 – Log C1/ log S2 – log S1

IV. FRACTALS & MUGHAL ARCHITECTURE

From the ancient times, architects thought about the philosophy of the straight lines and designed buildings and cities following the linear geometry but the contemporary architects started to indulge into the studies of the objects in space and started to study the link between architectural structures and elements of nature. They come to the conclusion that there are other types of objects which are not carrying the characters of linear geometry. They also started to think about the metaphysical properties of the buildings. The complex relationship between the order and geometry created another theory which is closer to the nature. Architects started to take interest in the fractal geometry in the buildings and landscapes [7].

The presence of fractals in architecture is neither a recent phenomenon nor a postmodern activity. It has been studied in the traditional settlements also. Fractal properties can be identified in mosques, temples, cathedrals, palaces, gardens etc. Fractals are very popular in the researchers due to the fractal characteristic such as rhythm, scale and progression as architecture is also concerned with the control of rhythm and scale. Architectural composition is related with progression of interesting forms. Fractal geometry provides a quantifiable tool for the blend of order and surprise [10], [12].

Mughals used reflections of the elements of nature in their buildings in the plans, elevations and in decorative patterns. These elements are symbols of fractals and can be shown even in the decorative patterns. Mughal buildings are in variety of structures such as palaces, mosques, tombs and gardens and they confirm the existence of fractals.

V. CASE STUDY OF WAZIR KHAN MOSQUE LAHORE

Wazir Khan Mosque is also one of those structures which are classic example of that period and possess all the characteristics of Mughal period. This mosque is showcase of balance, symmetry, rhythm, harmony, hierarchy and proportion. The depth of an arch, type of windows, floor and wall pattern are representation of self-similarity and iteration of fractals. The application of fractal geometry has also been adopted widely in designing stucco and fresco works to decor the interiors of the domes. Wazir Khan Mosque is located on the western side of the Wazir Khan Chowk (square) about 260 meters from Dehli gate it was constructed by Hakim Aliuddin in 1963 (1054 AH) This mosque is central part of an historic urban space including Shahi Hamam in the walled city of Lahore [1].

Wazir Khan Mosque is famous for its decorative elements developed during the Shah Jahan's period. It includes kashi-kari tile work, frescos and embellishments. The mosque also contains the cypress trees. This motif has been used first time in this mosque borrowed from Persia. The calligraphy done in this mosque is also the unique style.

A. Measurement of Fractal Dimension in Plan & Elevation

Fractal Geometry has been applied in Mughal Architecture in two dimension as well as in three dimensional spaces. Wazir Khan Mosque is classic example of Mughal Architecture and expresses occurrence of fractal geometry. The depth of an arch, types of windows, floor and Wall pattern all shows ordered repetition, radiated patterns and ornamentation based on geometry which are attributes of fractals. The plan of Wazir Khan Mosque clearly depicts the existence of fractals in the horizontal spatial divisions through its characteristics

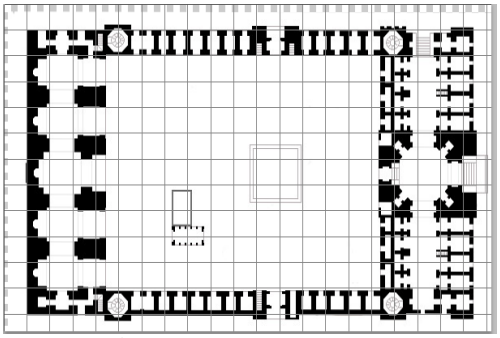

Fig. 6 Plan of Wazir Khan Mosque with grid size s1 = 20

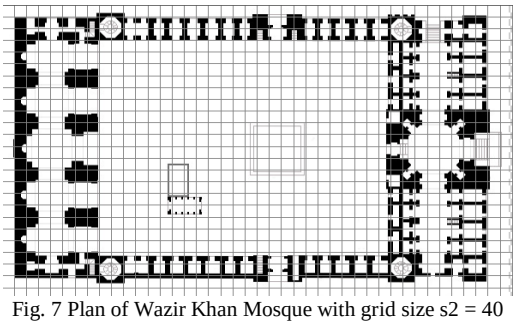

Fig. 7 Plan of Wazir Khan Mosque with grid size s2 = 40

Table 2 tabulation of Results with Box-counting method

Sr.no.	Grid Size	Occupied Boxes Count
1.	S1=20	C1=228
2.	S2=40	C2=882

D (2 to 1) = log 882 – log 228/ log 40 – log 20
= 2.94 – 2.35 / 1.62 – 1.30
= 0.59/0.32
= 1.84 means its fractal geometry

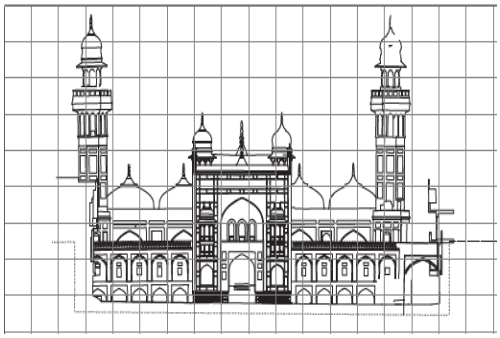

Fig. 8 Elevation of Wazir Khan Mosque Lahore with grid size s1= 15

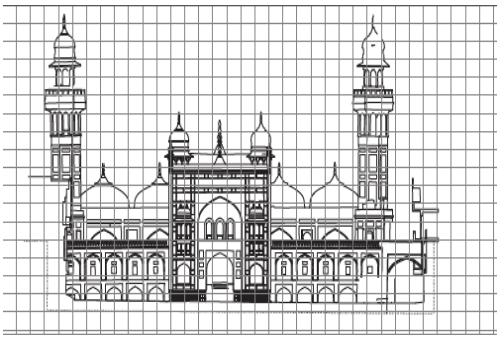

Fig. 9 Elevation of Wazir Khan Mosque Lahore with grid size s2= 30

Table 3 tabulation of Results with Box-counting method

Sr.no.	Grid Size	Occupied Boxes Count
1.	S1=15	C1=98
2.	S2=30	C2=350

D (2 to 1) = log 350 – log 98 / log 30-log 15
= 2.54 – 1.99/1.47 – 1.17
= 0.55 / 0.3
= 1.83 means it's a fractal geometry

The value of fractal dimension shows the level of intricacy and irregularity. The higher value of fractal dimension is due to the greater details in the architectural forms. It is generally observed that a fractal dimension of one to two is demonstrated in buildings. The process is complicated but it is a method to analyze the fractals in the Wazir khan Mosque Lahore it is also a fact that there is a need to further explore this area of research.

VI. CONCLUSIONS

i There is a high homogeneity and harmony in the façades of the Wazir Khan mosques selected from the Mughal architecture as there was a high approximation in the values of the fractal properties of the building parts in spite of the change in the measure of these parts, i. e. the small parts are similar to the bigger ones in terms of the fractal geometry.

ii The formal characteristics that are related to the surface level to form the façade in the selected mosque achieved the unity and diversity and that made this building rich in their formations because unity was fulfilled through the high approximation in the values of the fractal dimension for the parts of the building façade.

iii For the diversity, it was fulfilled through the iteration investigated for this building with smaller scales away from routine resulting from the monotonous repetition of these parts. The façades of the Wazir Khan mosque in the Mughal architecture are characterized by an architectural language that implicates a delicate balance between the repetition and diversity and the diversity away from chaos and randomness and this results in accomplishing the positive system that delights the recipient of this type of architecture.

iv The façade of the selected building is all characterized with fractal and approximation on the level of the façade as a whole or the parts of the façade. This indicates that there is a high homogeneity amongst the façades selected.

v The façade of the selected building possess the balanced natural features in addition to possessing the rich and deep structure that can be referred to enrich the current architectural products.

vi Through reviewing several samples in the decorative Architecture, we can say that the mosque architecture is mainly characterized with the self-similarity in terms of the buildings façades.

vii Using the fractal characteristics that were concluded in this research will contribute to generate new buildings that are related to the historical and cultural sense of the M architecture without having to copy and imitate the elements of this architecture.

viii There is a close relationship between the concepts of the fractal geometry and architecture, so the fractal geometry can be employed and its concepts and characteristics can be used to analyze the architectural buildings and studying their complexities.

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